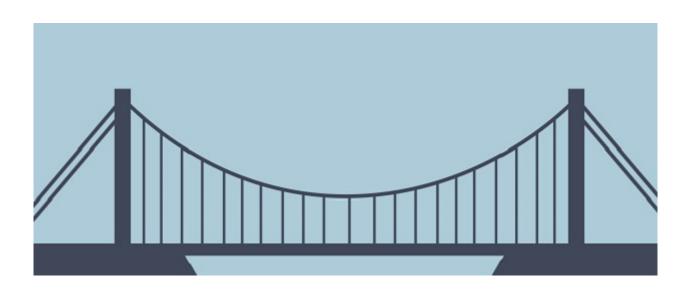


The Bridge to A level

Problem Solving Solutions





Solving quadratic equations 1

Question 1

A number and its reciprocal add up to $\frac{26}{5}$.

Form and solve an equation to calculate the number.

$$x + \frac{1}{x} = \frac{26}{5}$$

$$x^{2} + 1 = \frac{26x}{5}$$

$$5x^{2} + 5 = 26x$$
(M1)

$$5x^2 + 5 = 26x$$

$$5x^2 - 26x + 5 = 0 \tag{M1}$$

$$(5x - 1)(x - 5) = 0$$

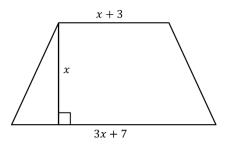
$$x = \frac{1}{5} \text{ or } 5 \text{ so the number is } 5$$
 (A1)

(3)

Question 2

The diagram shows a trapezium.

Diagram **NOT** accurately drawn



All the measurements are in centimetres.

The area of the trapezium is 16 cm².

a) Show that $2x^2 + 5x - 16 = 0$

Area =
$$\frac{1}{2}x((x+3) + (3x+7))$$

 $16 = \frac{1}{2}x(4x+10)$
 $16 = 2x^2 + 5x$
 $2x^2 + 5x - 16 = 0$

(1)

b) Work out the value of x to 1 decimal place.

$$\chi = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -16}}{2 \times 2}$$
 (M1)

x = 1.842329219...or $x = -4.342329219 \dots$

$$x = 1.8 \tag{A1}$$

 $x = \dots (2)$



Two numbers have a product of 44 and a mean of 7.5.

Use an algebraic method to find the numbers.

You must show all of your working.

1st number: x then 2nd number:
$$\frac{44}{x}$$

$$\frac{x + \frac{44}{x}}{2} = 7.5 \qquad (M1)$$

$$x + \frac{44}{x} = 15$$

$$x^2 + 44 = 15x$$

$$x^2 - 15x + 44 = 0 \qquad (M1)$$

$$(x - 11)(x - 4) = 0 \qquad (M1)$$

$$x = 11 \text{ or } 4 \qquad \text{so the numbers are } 11 \text{ and } 4 \qquad (A1)$$

(4)



2 <u>Changing the subject</u>

Question 1

The surface gravity of a planet is given by $g=rac{GM}{r^2}$ where

M = Mass of the planet r = radius of the planet G = gravitational constant = $6.67x10^{-11}$

The surface gravity of Earth is $9.807 \,\mathrm{m/s^2}$ and the mass of Earth is $5.98 \,\mathrm{x} \, 10^{24} \mathrm{kg}$.

Find the radius of Earth in kilometres correct to 3 significant figures.

$$r = \sqrt{\frac{GM}{g}}$$
 M1

$$r = \sqrt{\frac{6.67x10^{-11}x5.98x10^{24}}{9.807}}$$
 M1

(4)

Question 2

In a parallel circuit, the total resistance is given by the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Make R_1 the subject of the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1xR_2 = RxR_2 + RxR_1$$
 M1 Multiplying out denominators

$$R_1xR_2 - RxR_1 = RxR_2$$
 M1 Collecting R_1 terms together

$$R_1(R_2 - R) = RxR_2$$
 M1 Factorising

$$R_1 = \frac{RxR_2}{(R_2 - R)}$$
 A1

(4)



Show that $\frac{1}{\frac{1}{x}+1} = \frac{x}{x+1}$

$$\frac{1}{\frac{1}{x}+1} = \frac{1}{\frac{x+1}{x}}$$

$$=\frac{1}{1} \div \frac{x+1}{x}$$

M1

$$= \frac{1}{1} \times \frac{x}{x+1}$$

 $= \frac{x}{x+1} \text{ as required}$ A1

(2)



3 Simultaneous equations

Question 1

Sarah intended to spend £6.00 on prizes for her class but each prize cost her 10p more than expected, so she had to buy 5 fewer prizes.

Calculate the cost of each prize.

Let x be no. of prizess & y be the price of each prize

$$xy = 600$$
 => $x = \frac{600}{y}$ (M1)

$$(x-5)(y+10) = 600$$

 $xy-5y+10x-50 = 600$ (M1)

$$\frac{600y}{y} - 5y + \frac{6000}{y} - 50 = 600 \tag{M1}$$

$$5y + 50 + \frac{6000}{y} = 0$$

$$5y^2 + 50y + 6000 = 0$$

$$y^2 + 10y + 1200 = 0$$

$$(y + 40)(y - 30) = 0$$
(M1)

y is a price so y = 30Cost of each prize = y + 10 = 40p (A1)

.....(5)

Question 2

Arthur and Florence are going to the theatre.

Arthur buys 6 adult tickets and 2 child tickets and pays £39.

Florence buys 5 adult tickets and 3 child tickets and pays £36.50.

Work out the costs of both adult and child tickets.

$$6A + 2C = 39$$
 x3 $18A + 6C = 117$

$$5A + 3C = 36.50$$
 x2 $10A + 6C = 73$ (M1 both correct)

Subtracting equations: 8A = 44 M1

A = 5.5

Substitute: (6x5.5)+2C = 39 M1

C = 3

Adult ticket = £5.50 A1 Child ticket = £3 A1

(5)

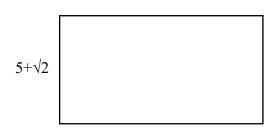


4 <u>Surds</u>

Question 1

Calculate the area of each shape giving your answers in the form $a+b\sqrt{2}$

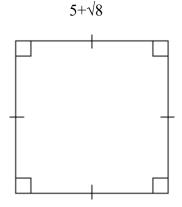
a) $11-\sqrt{2}$



$$(11 - \sqrt{2})(5 + \sqrt{2})$$

= $55 - 5\sqrt{2} + 11\sqrt{2} - 2$ M1
= $53 + 6\sqrt{2}$ A1

b)



$$(5+\sqrt{8})(5+\sqrt{8}) = 25+10\sqrt{8}+8$$
 M1
= $33+10\sqrt{8}$ M1
= $33+10\sqrt{4}\sqrt{2}$
= $11+20\sqrt{2}$ A1

(3)

(2)

Question 2

Colin has made several mistakes in his 'simplifying surds' homework. Explain his error and give the correct answer.

i)
$$4\sqrt{3} \times 5\sqrt{12} = 20\sqrt{36}$$

C1 for a valid explanation A1 for 120

(2)



The area of a triangle is 20cm³. The length of the base is V8cm. Work out the perpendicular height giving your answer as a surd in its simplest form.

$$20 = \frac{\sqrt{8 \times x}}{2}$$

$$40 = \sqrt{8} \times x$$

$$\frac{40}{\sqrt{8}} = x$$
 M1

$$\frac{40}{\sqrt{8}} = x$$
$$x = \frac{40}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}}$$

$$=\frac{40\sqrt{8}}{8}$$

$$=5\sqrt{8}$$
 M1

$$=10\sqrt{2}$$
 A1

(3)



5 **Indices**

Question 1

Lowenna says that $27^{-1/3} \times 64^{2/3} = 48$

Is Lowenna correct? You must show all of your working.

$$27^{-1/3} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$$

$$64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$$

$$\frac{1}{3} \times 16 = \frac{16}{3} \neq 5$$
 so Lowenna is not correct

Question 2

Which one of these indices is the odd one out? Circle your answer and give reasons for your choice.



$$8^{-\frac{1}{3}}$$

B1 for correct answer circled

C1 for correct explanation, with at least two indices evaluated

Question 3

(2)

(4)

Find values for a and b that make this equation work

$$a^{\frac{1}{2}} = b^{\frac{1}{3}}$$

$$a = 16$$
 and $b = 64$ (A1)

a = 16 and b = 64 (A1) (note: other solutions possible)

(1)

Question 4

Write 25 as a power of 125 i)

$$125^{\frac{2}{3}}$$
 (A1)

(1)

ii) Write 4 as a power of 32

$$32^{\frac{2}{5}}$$
 (A1)

(1)

iii) Write 81 as a power of 27

$$27^{\frac{4}{3}}$$
 (A1)

(1)



6 Properties of Lines

Question 1

(a) (a) Write down the gradient of the line
$$2y - 4x = 5$$
. m = 2 (A1)

(1)

(b) Write down the equation of a line parallel to 3y = 7 - 4x.

$$y = -4x/3 + k \quad \text{for any } k \tag{A1}$$

(1)

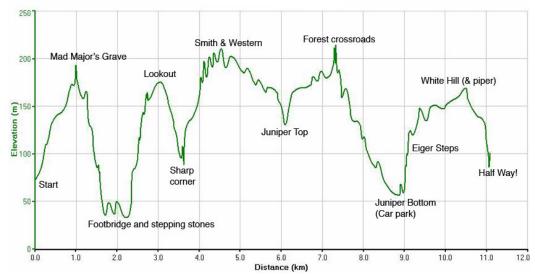
(c) Write down the equation of a line with gradient ½ and y-intercept of 6.

$$y = \frac{1}{2}x + 6$$
 (A1)

(1)

Question 2

Here is the profile of the first half of a fell running race.



(a) Work out the approximate gradient of the race from the start to Mad Major's Grave

$$\frac{up}{along} = \frac{195-75}{1000-0}$$
 (M1) accept approx. values m = 0.12

(2)

(b) The most dangerous part of the race is from Mad Major's Grave to the Footbridge. Why do you think this might be?

It is the steepest part of the course (and it is downhill) (C1)

(1)

(c) Work out an estimate for the average ascent for the first four uphill sections of the race.

Section
$$1 = \frac{up}{along} = \frac{195-75}{1000} = 0.12$$
 Section $2 = \frac{up}{along} = \frac{175-30}{1000} = 0.145$
Section $3 = \frac{up}{along} = \frac{210-90}{1000} = 0.12$ Section $4 = \frac{up}{along} = \frac{220-130}{1000} = 0.09$

Method to find four ascents using graph

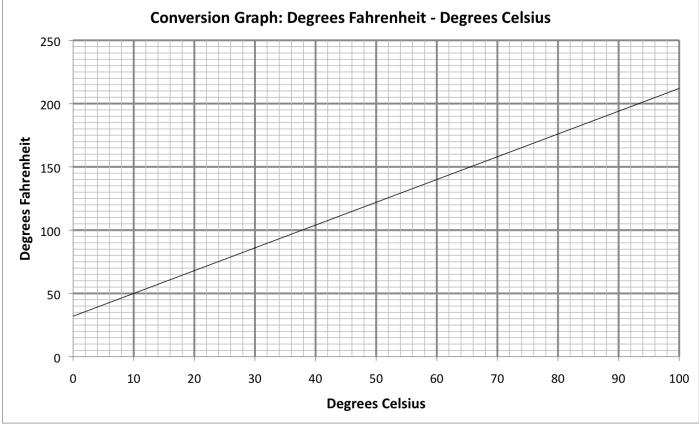
Average of 0.12, 0.145, 0.12, and 0.09 = 0.11875 Km (A1)ft

(2)

Question 3



Here is a graph used to convert degrees Celsius (C) and degrees Fahrenheit (F).



The equation of the straight line is given by F = mC + aCalculate the values of m and a

Method to find gradient =
$$\frac{up}{along} = \frac{18}{10}$$
 (M1)

m = 1.8 or 9/5a = 32 (A1) both correct



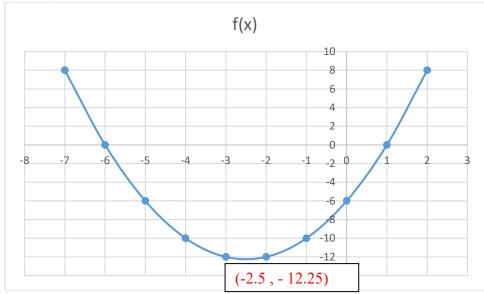
7 <u>Sketching curves</u>

Question 1

Sketch the graph of $f(x) = x^2 + 5x - 6$, showing the co-ordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

$$(x + 2.5)^2 - 6.25 - 6$$

$$(x + 2.5)^2 - 12.25$$

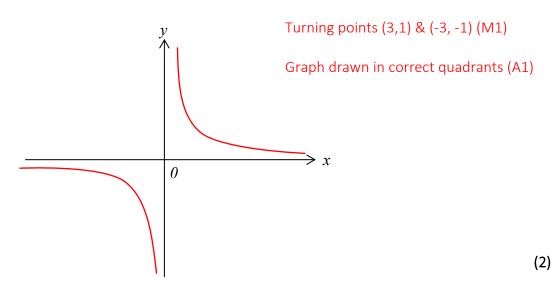


- B1 Correct shape, right way up
- B1 Min point (-2.5, -12.25) marked
- B1 -6 marked
- B1 x=-6 and x=1 marked

(5)

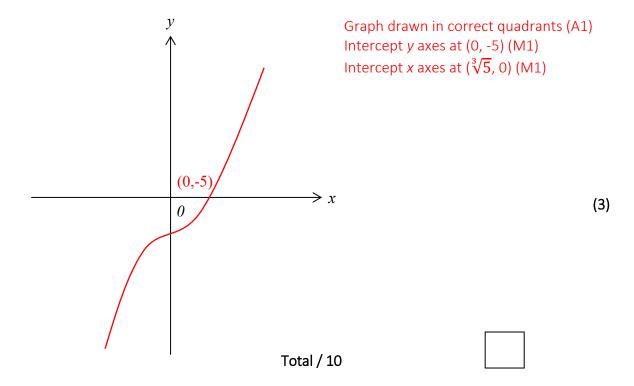
Question 2

a) On the axes sketch the graph of $y = \frac{3}{x}$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.





b) On the axes sketch the graph of $y = x^3 - 5$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.





8 <u>Transformation of functions</u>

Question 1

Here is a sketch of f(x).

The coordinates of P are (0,-2)

Sketch the graphs after the following translations and reflections, and state the coordinates of P':

a)
$$g(x) = f(x) + 1$$

$$P' = (0, -1)$$

b)
$$h(x) = f(x - 2)$$

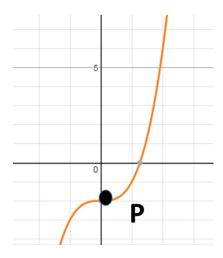
$$P' = (2, -2)$$

c)
$$j(x) = -f(x)$$

$$P' = (0, 2)$$

d)
$$k(x) = f(-x)$$

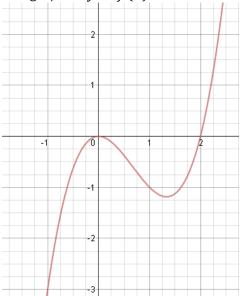
$$P' = (0, -2)$$



(4)

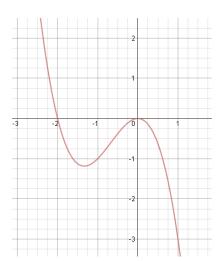
Question 2

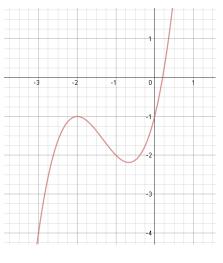
The graph of y = f(x) is shown below.



Below each sketch, write down the equation of the transformed graph







$$y =f(-x).....B1.....$$

$$y =f(x + 2) - 1$$
 B1

(2)

Question 3

The equation of a curve is y = f(x) where $f(x) = x^2 - 4x + 5$ C is the minimum point of the curve.

(a) Find the coordinates of C after the transformation f(x + 1) + 2.

 $f(x) = (x - 2)^2 + 1$

Before transformation C is (2,1) M1 After transformation C is (1,3) A1

(.....) **(2)**

(b) Hence, or otherwise, determine if f(x-3)-1=0 has any real roots. Give reasons for your answer.

Min point for f(x-3) - 1 is at (5, 0)

M1

Hence it has a single, repeated root at x = 5

L

A1

(2) Total / 10



Pythagoras' theorem and Trigonometric ratios 9

Question 1

ABCDEFGH is a cuboid

AE = 5cm

AB = 6cm

BC = 9cm

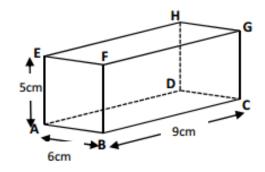


Diagram NOT drawn accurately

(a) Calculate the length of AG. Give your answer correct to 3 significant figures.

AG =
$$V(6^2 + 9^2 + 5^2) = \sqrt{(36 + 81 + 25)} = V142 = 11.9$$
cm

(1)

(b) Calculate the size of the angle between AG and the face ABCD. Give your answer correct to 1 decimal place.

$$Sin\theta = 5 \div \sqrt{142} = 0.41959$$
 (M1 ft from (a))
 $\Theta = 24.8^{\circ}$ (A1)

 $\sqrt{142}$

Question 2

A piece of land is the shape of an isosceles triangle with sides 7.5m, 7.5m and 11m. Turf can be bought for £11.99 per 5m² roll.

How much will it cost to turf the piece of land?

$$V(7.5^2 - 5.5^2) = 5.10$$
m length of land (M1)

Area of land =
$$11 \times 5.1 \div 2 = 28.05 \text{m}^2$$
 (M1)

(3)

(3)

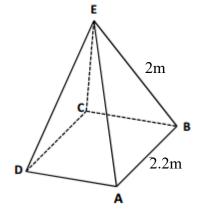


Ben is 1.62m tall.

The tent he is considering buying is a square based pyramid.

The length of the base is 3.2m.

The poles AE, CE, AE and BE are 2m long.



Ben wants to know if he will be able to stand up in the middle of the tent. Explain your answer clearly.

$$DB = V(2.2^2 + 2.2^2) = 3.1m (M1)$$

Height =
$$V(2^2 - 1.55^2) = 1.5975 \text{m}$$
 (M1)

Ben will be able to stand up in the tent (A1)

.....

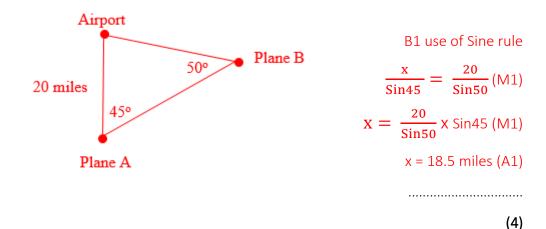
(3)



10 Sine / Cosine Rule

Question 1

Plane A is flying directly toward the airport which is 20 miles away. The pilot notice a second plane, B, 45° to her right. Plane B is also flying directly towards the airport. The pilot of plane B calculates that plane A is 50° to his left. Based on that information how far is plane B from the airport? Give your answer to 3 significant figures.



Question 2

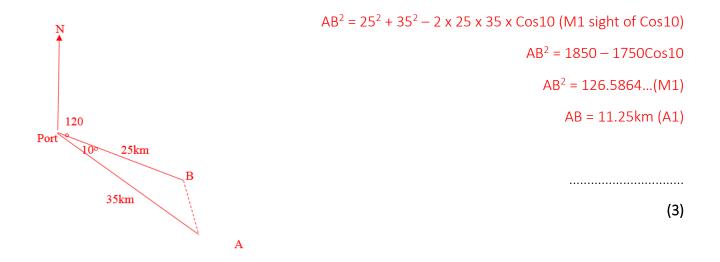
Two ships, A and B, leave the same port at the same time.

Ship A travels at 35km/h on a bearing of 130°.

Ship B travels at 25km/h on a bearing of 120°.

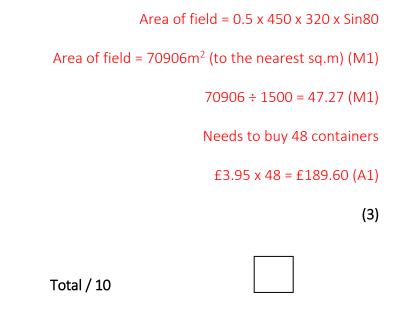
Calculate how far apart the ships are after 1 hour.

Give your answer correct to two decimal places.





A farmer has a triangular field. He knows one side measures 450m and another 320m. The angle between these two sides measures 80°. The farmer wishes to use a fertiliser that costs £3.95 per container which covers 1500m². How much will it cost to use the fertiliser on this field?





11 <u>Inequalities</u>

Question 1

A new cylindrical tube of snacks is being designed so that its height is 3 times its radius and its volume must be less than 20 times its radius.

Create an inequality and find possible values for the radius.

$$\pi r^2$$
. $h < 20r$ $h =$

$$\pi r^2 . 3r < 20r$$

$$3\pi r^3 < 20r \tag{M1}$$

$$3\pi r^2 < 20$$
 (M1) $r^2 < \frac{20}{3\pi}$

$$r < \sqrt{\frac{20}{3\pi}} \tag{A1}$$

Note; cannot have a negative length.

(3)

Question 2

A base jumper is going to jump off a cliff that is 50m tall, the distance she travels downwards is given by the equation

$$d = 4.9t^2$$
 where $t = time of flight$
and $d = distance travelled$

A video camera is set-up to film her between 20m and 10m above the ground. Calculate the time period after the jumper jumps that filming taking place.

20m above ground = 30m downwards

10m above ground = 40m downwards

$$So\ 30 \le d \le 40 \tag{M1}$$

$$30 \le 4.9t^2 \le 40$$

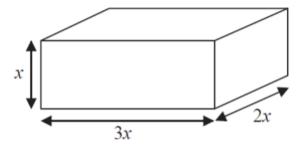
$$\frac{30}{4.9} \le t^2 \le \frac{40}{4.9} \tag{M1}$$

$$2.47 \le t \le 2.86 \, s$$
 (A1)

(3)



The total volume of the box is less than 1 litre. Given that all lengths are in cm and that x is an integer, Show that the longest side is less than 18cm.



Volume = $6x^2$ M1

 $1l = 1000 \text{cm}^3 \text{ therefore}$ $6x^2 < 1000$ M1

 $x^3 < 33.3$

 $\sqrt[3]{333}$. 3 = 6.93 M1

X < 6.93 and an integer so a max value of 6 so 3x < 18

(4)



12 Algebraic proof

Question 1

Katie chooses a two-digit number, reverses the digits, and subtracts the smaller number from the larger.

For example

$$42 - 24 = 18$$

She tries several different numbers and finds the answer is never a prime number.

Prove that Katie can never get an answer that is a prime number.

My numbers are

10a + b and 10b + a

10a + b - (10b + a)

= 10a - 10b + b - a

= 9a - 9b

= 9 (a - b)

The answer is always a multiple of 9

- Attempts to write an expression for the first number
- Writes the correct expression for the first number
- Writes the correct expression for the second number
- Attempts to subtract the expressions
- Simplifies the result
- Factorises
- Makes the statement

Question 2

Here are the first 5 terms of an arithmetic sequence

1 6 11

16 21

Prove that the difference between the squares of any 2 terms is always a multiple of 5.

nth term = 5n-4

(n+1)th term = 5(n+1)-4 = 5n+1

Square nth term = $(5n-4)^2 = 25n^2 + -40n + 16$

Square (n+1)th term = $(5n+1)^2 = 24n^2+10n+1$

Difference = $(24n^2+10n+1) - (25n^2+-40n+16) = 50n-15$

Factorise = 5(10n-3) which is a multiple of 5

(6)

(4)



13 **Vectors**

Question 1.

OAB is a triangle

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

(a) Find the vector AB in terms of a and b

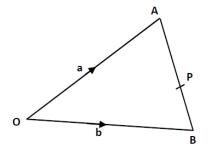


Diagram NOT drawn accurately

$$AB = -\mathbf{a} + \mathbf{b} \text{ or } \mathbf{b} - \mathbf{a} \text{ (B1)}$$

..... **(1)**

P is the point on \overrightarrow{AB} such that AP: PB = 3:2

Show that $\overrightarrow{OP} = \frac{1}{5} (2\mathbf{a} + 3\mathbf{b})$ (b)

(c)
$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

(M1) vector equation shown or implied in working

(d)
$$\overrightarrow{OP} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

(M1) using equation from part (a)

(e)
$$\overrightarrow{OP} = \mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$$

(f)
$$\overrightarrow{OP} = \frac{2}{5} \mathbf{a} + \frac{3}{5} \mathbf{b} = \frac{1}{5} (2\mathbf{a} + 3\mathbf{b})$$

(f) $\overrightarrow{OP} = \frac{2}{5} \mathbf{a} + \frac{3}{5} \mathbf{b} = \frac{1}{5} (2\mathbf{a} + 3\mathbf{b})$ (M1) simplified expression then factorisation clearly shown

(3)

Question 2.

OABC is a parallelogram.

X is the midpoint of OB

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

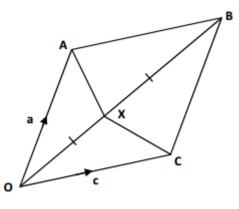


Diagram NOT drawn accurately

(a) Find the vector \mathbf{OX} in terms of \mathbf{a} a

$$\overrightarrow{OX} = \frac{1}{2} (\mathbf{a} + \mathbf{c}) \quad (B1)$$
.....(1)



(b) Find the vector \overrightarrow{XC} in terms of \mathbf{a} and \mathbf{c} .

$$\overrightarrow{XC} = \overrightarrow{XO} + \overrightarrow{OC} = -\frac{1}{2} (\mathbf{a} + \mathbf{c}) + \mathbf{c} \quad (M1)$$

$$\overrightarrow{XC} = -\frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{c} \quad \text{or} \quad \frac{1}{2} (\mathbf{c} - \mathbf{a}) \quad (A1)$$

$$(2)$$

Question 3

PQRS is a parallelogram. M is the midpoint of RS N is the midpoint of QR PQ = 2aPS = 2b

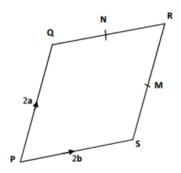


Diagram NOT drawn accurately

Use vectors to proof that the line segments SQ and MN are parallel.

$$SQ = -2b + 2a (M1)$$

$$\overrightarrow{MN} = \mathbf{a} - \mathbf{b} \ (M1)$$

$$\overrightarrow{SQ} = 2MN$$
 therefore parallel (A1)

(3)



14 <u>Probability</u>

Question 1

Max has an empty box.

He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1:3.

Julie takes at random 2 counters from the box.

The probability that she takes 2 red counters is $\frac{19}{316}$.

How many red counters did Max put in the box?

For process to start to solve. E.g. use of
$$x$$
 and $3x$ M1

To form fractions for each probability. E.g. $\frac{x}{4x}$ and $\frac{3x}{4x}$, $\frac{x-1}{4x-1}$ M1

Process to form equation e.g. $\frac{x}{4x} \times \frac{x-1}{4x-1} = \frac{19}{316}$ M1

Process to eliminate fractions and reduce equation to linear form E.g. $316x - 316 = 304x - 76$

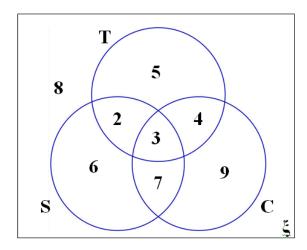
(5)



The Venn diagram shows the ice-cream flavours chosen by a group of 44 children at a party.

The choices are strawberry (S), choc-chip (C) and toffee (T).

A child is picked at random.



Work out:

$$\frac{18}{44} = \frac{9}{22}$$

A1

(1)

(b) $P(T \cup C \mid C)$

$$\frac{\frac{7}{23}}{\text{(Allow M1 for }\frac{7}{n} \text{ or }\frac{n}{23}\text{)}}$$

M2

(2)

(c)
$$P(C|S \cup T)$$

$$\frac{\frac{14}{27}}{\text{(Allow M1 for } \frac{14}{n} \text{ or } \frac{n}{27})}$$

M2

.....

(2)



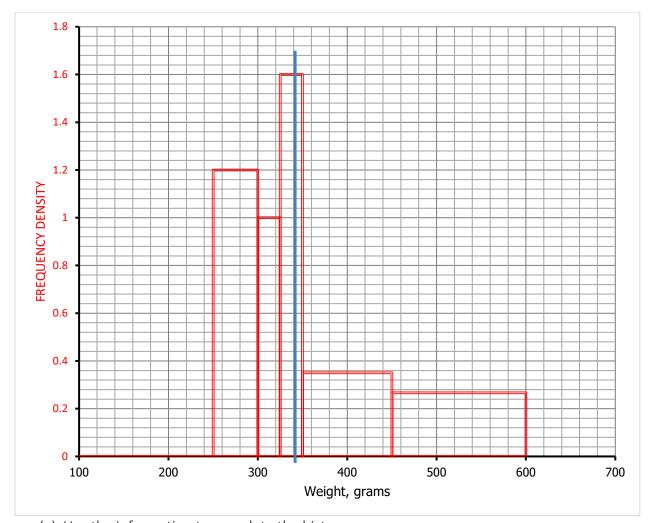
15 <u>Statistics</u>

Question 1

The table and histogram show the weights of some snakes.

Weight, grams			Frequency
250	< X ≤	300	60
300	< x <	325	25
325	< x <	350	40
350	< x <	450	35
450	< x <	600	40
		Total	200

Class	Freq.	
Width	Density	
50	1.2	
25	1.0	
25	1.6	
100	0.4	
150	0.2666666	



(a) Use the information to complete the histogram

Middle bar frequency = 40 class width = 25 frequncy density = 40/25 = 1.6M1Draw in scale M1 Draw rest of bars correctly A1



(b) Calculate an estimate for the median 200 snakes, median at 201/2 = 100.5th (condone 100th)

$$200 - (40 + 35 + (25))$$

M1

$$350 - 15.625 = (334.375)$$

.....334 grams......A1.....(2)

28

28

28

Question 2

Sarah played 15 games of netball. Here are the number of goals she scored in each game.

22

24

25

25

26

21

17 17 17

20

19

a) Draw a boxplot to show this information

18

Smallest value = 17

Largest value = 28

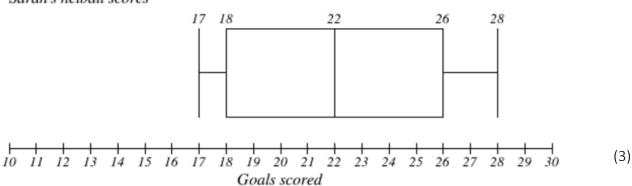
Median = 22

LQ = 18

UQ = 26

(M1M1 for calculations and A1 for graph)

Sarah's netball scores



b) Lucy plays in the same 15 games of netball. The median number of points Lucy scores is 24. The interquartile range of these points is 10 and the range of these points is 17.

Who is the better player, Sarah or Lucy?

You must give a reason for your answer.

Comparison numbers for Sarah: 22, 8 and 11

Sarah is more consistent as she has a smaller IQR and smaller range.

M1

Lucy scores more goals on average as she has a higher median.

M1

(2)