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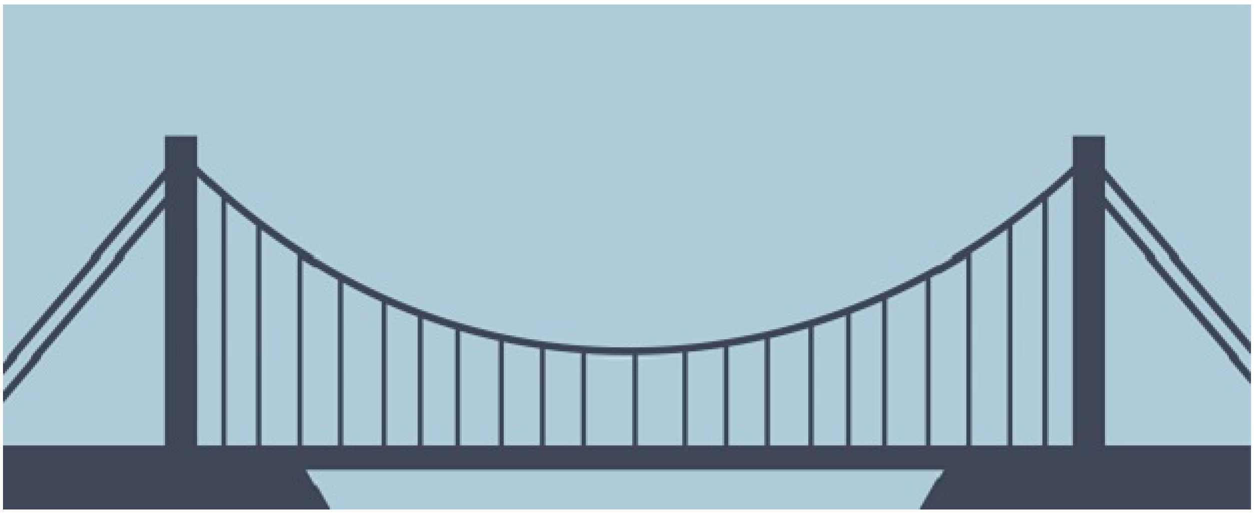
The Bridge to A level

Diagnosis Worked Solutions



1 Solving quadratic equations

Question 1

Solve $x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

$x = -2$ or -4

(2)

Question 2

Solve the equation $y^2 - 7y + 12 = 0$

Hence solve the equation $x^4 - 7x^2 + 12 = 0$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0 \rightarrow \underline{y=3}$ or $\underline{y=4}$

$x^4 - 7x^2 + 12 = 0 \rightarrow$ let $x^2 = y$

$(x^2)^2 - 7x^2 + 12 = 0 \rightarrow y^2 - 7y + 12 = 0 \rightarrow y = 3$ or $y = 4$

$\rightarrow x^2 = 3$ or $x^2 = 4$

$\rightarrow \underline{x = \pm\sqrt{3} \text{ or } x = \pm 2}$

(4)

Question 3

(i) Express $x^2 - 6x + 2$ in the form $(x-a)^2 - b$

$x^2 - 6x + 2 = (x-3)^2 - 9 + 2$

$= \underline{(x-3)^2 - 7}$

(3)

(ii) State the coordinates of the minimum value on the graph of $y = x^2 - 6x + 2$

Minimum point of $x^2 - 6x + 2$ is therefore $\underline{(3, -7)}$

(1)

Total / 10

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2 Changing the subject

Question 1

Make v the subject of the formula $E = \frac{1}{2} mv^2$

$$E = \frac{1}{2} mv^2$$

$$\Rightarrow 2E = mv^2$$

$$\Rightarrow \frac{2E}{m} = v^2$$

$$\pm\sqrt{\frac{2E}{m}} = v$$

(3)

Question 2

Make r the subject of the formula $V = \frac{4}{3} \pi r^2$

$$V = \frac{4}{3}\pi r^3$$ get rid of fractions

$$3V = 4\pi r^3$$ make r^3 the subject.

$$\frac{3V}{4\pi} = r^3$$ cube both sides

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

(3)

Question 3

Make c the subject of the formula $P = \frac{c}{c+4}$

$$P = \frac{c}{c+4}$$ Get rid of fractions

$$\Rightarrow P(c+4) = c$$ Expand brackets

$$\Rightarrow Pc + 4P = c$$ Get terms with c on L.H.S., other terms on R.H.S.

$$Pc + 4P - c = 0$$

$$Pc - c = -4P$$ Factorise.

$$c(P-1) = -4P$$ Divide

$$c = \frac{-4P}{P-1} \quad \left(= \frac{4P}{1-P}\right)$$

(4)

Total / 10

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3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines y = 3x + 1 and x + 3y = 6

$y = 3x+1$ and $x+3y=6$

$x + 3(3x+1) = 6$

$x + 9x + 3 = 6$

$10x = 3$

$x = \frac{3}{10}$

$y = 3\left(\frac{3}{10}\right)+1$

$= \frac{9}{10} + 1$

$= 1\frac{9}{10}$

$\left(\frac{3}{10}, 1\frac{9}{10}\right)$ or $(0.3, 1.9)$

(3)

(3)

Question 2

Find the coordinates of the point of intersection of the lines 5x + 2y = 20 and y = 5 – x

(ii) $5x + 2y = 20$ & $y = 5 - x$.

Solving simultaneously

$5x + 2(5-x) = 20$ M1

$\Rightarrow 5x + 10 - 2x = 20$

$\Rightarrow 3x = 10$

$\Rightarrow x = 10/3$, $y = 5 - 10/3 = 5/3$

Pt. of intersection is $(10/3, 5/3)$ A2

Note – if you round these fractions to decimals (10.3, 1.7) you lose a mark.

(3)

Question 3

Solve the simultaneous equations

$x^2 + y^2 = 5$ $\qquad$ $y = 3x + 1$

Sub in $y = 3x+1$ into equation 2.

$x^2 + (3x+1)^2 = 5$

$x^2 + (3x+1)(3x+1) = 5$

$x^2 + 9x^2 + 3x + 3x + 1 = 5$

$10x^2 + 6x + 1 = 5$

$10x^2 + 6x - 4 = 0$

$(\div 2)$

$5x^2 + 3x - 2 = 0$

$(5x-2)(x+1) = 0$

$x = \frac{2}{5}$ or $x = -1$

when $x = \frac{2}{5}$

$y = \left(3 \times \frac{2}{5}\right) + 1$

$= \frac{6}{5} + \frac{5}{5} = \frac{11}{5}$

when $x = -1$

$y = (3 \times -1) + 1$

$= -3 + 1$

$= -2$

(4)

Total / 10

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4 Surds

Question 1

(i) Simplify $(3 + \sqrt{2})(3 - \sqrt{2})$

$(3+\sqrt{2})(3-\sqrt{2})$

$= 3^2 + 3\sqrt{2} - 3\sqrt{2} - (\sqrt{2})^2$

$= 9 - 2$

$= 7$

(2)

(ii) Express $\frac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational

$\frac{(1+\sqrt{2})}{(3-\sqrt{2})} = \frac{(1+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

$= \frac{3+\sqrt{2}+3\sqrt{2}+(\sqrt{2})^2}{7}$

$= \frac{3+4\sqrt{2}+2}{7}$

$= \frac{5}{7} + \frac{4}{7}\sqrt{2}$

To rationalise a denominator of form $(x+\sqrt{y})$ multiply top + bottom by $(x-\sqrt{y})$

(3)

Question 2

(i) Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

(i) $5\sqrt{8} + 4\sqrt{50}$

$= 5\sqrt{4}\sqrt{2} + 4\sqrt{25}\sqrt{2}$

$= 5\times2\sqrt{2} + 4\times5\sqrt{2}$

$= 10\sqrt{2} + 20\sqrt{2}$

$= 30\sqrt{2}$

Always look for square number factors

(2)

(ii) Express $\frac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational

$\frac{\sqrt{3}}{6-\sqrt{3}} = \frac{\sqrt{3}}{6-\sqrt{3}} \times \frac{(6+\sqrt{3})}{(6+\sqrt{3})}$

$= \frac{\sqrt{3}\times6 + \sqrt{3}\sqrt{3}}{6^2 - (\sqrt{3})^2}$

$= \frac{6\sqrt{3} + 3}{36-3}$

$= \frac{3+6\sqrt{3}}{33}$

$= \frac{3}{33} + \frac{6}{33}\sqrt{3}$

$= \frac{1}{11} + \frac{2}{11}\sqrt{3}.$

(3)

Total / 10

5 Indices

Question 1

Simplify the following

(i) a^0 (1)

(ii) $a^6 \div a^{-2}$ (1)

(iii) $(9a^6b^2)^{-0.5}$ (3)

(i) $a^0 = 1$

(ii) $a^6 \div a^{-2} = a^{6--2}$

$= a^8$

(iii) $(9a^6b^2)^{-\frac{1}{2}} = (3^2a^6b^2)^{-\frac{1}{2}}$

$= 3^{-1}a^{-3}b^{-1}$ $\left(= \frac{1}{3a^3b}\right)$

Question 2

(i) Find the value of $\left(\frac{1}{25}\right)^{-0.5}$ (2)

(ii) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$ (3)

i) $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = (25)^{\frac{1}{2}} = \sqrt{25} = \pm 5$

ii) $\frac{(2x^2y^3z)^5}{4y^2z} = \frac{2^5x^{10}y^{15}z^5}{2^2y^2z^1}$

$= 2^{5-2}x^{10}y^{15-2}z^{5-1}$

$= 2^3x^{10}y^{13}z^4 = 8x^{10}y^{13}z^4$

Total / 10

6 Properties of Lines

Question 1

A (0,2), B (7,9) and C (6,10) are three points.

(i) Show that AB and BC are perpendicular

Grad of AB = $\frac{9-2}{7-0}$ = 1

Grad of BC = $\frac{10-9}{6-7}$ = -1

Product of gradients = 1 x -1 = -1 → AB and BC perpendicular

(3)

(ii) Find the length of AC

$(6-0)^2 + (10-2)^2 = AC^2$
AC = 10

(2)

Question 2

Find, in the form y = mx + c, the equation of the line passing through A (3,7) and B (5,-1).
Show that the midpoint of AB lies on the line x + 2y = 10

$m = \frac{-1-7}{5-3} = \frac{-8}{2} = -4$

$y = -4x + c$

Substitute in (3,7) [5,-1] would do equally as well

$7 = -4 \times 3 + c$
$\Rightarrow 19 = c$

$\Rightarrow y = -4x + 19$

Midpoint of AB = (4, 3)

Sub. in to $x + 2y = 10$ & show that equation is true.

$4 + 2 \times 3 = 4 + 6 = 10$ TRUE.

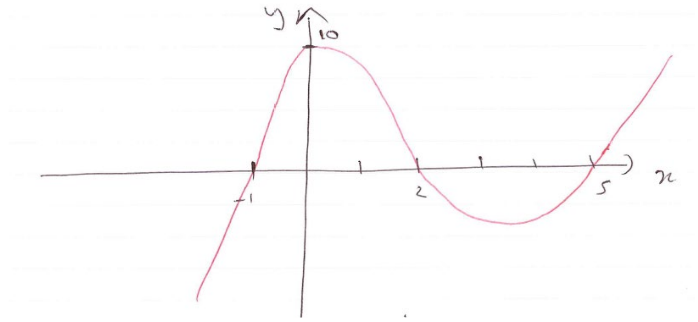
(5)

Total / 10

7 Sketching curves

Question 1

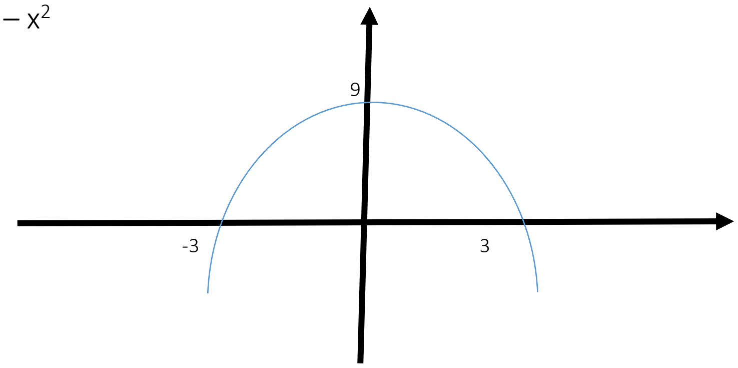
In the cubic polynomial f(x), the coefficient of x^3 is 1. The roots of f(x) = 0 are -1, 2 and 5.
Sketch the graph of y = f(x)



(3)

Question 2

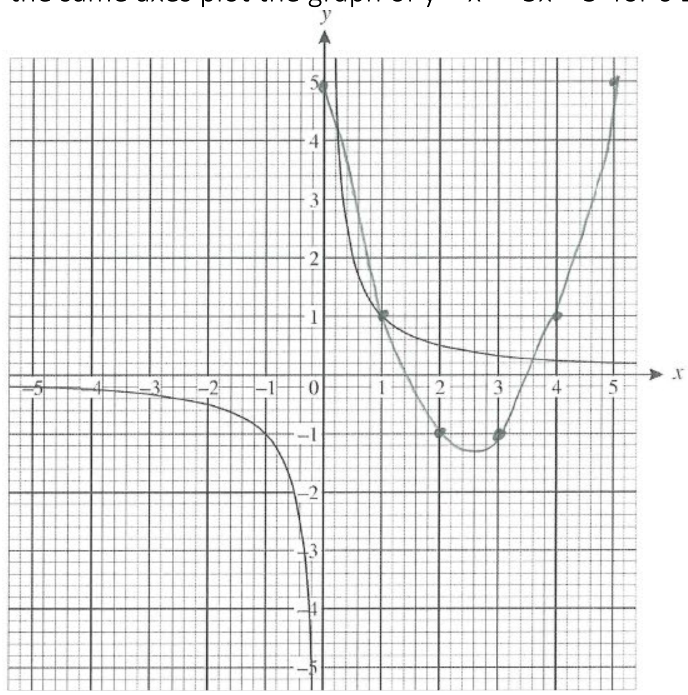
Sketch the graph of $y = 9 - x^2$



(3)

Question 3

The graph below shows the graph of $y = \frac{1}{x}$
On the same axes plot the graph of $y = x^2 - 5x + 5$ for $0 \le x \le 5$



x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$-5x$	0	-5	-10	-15	-20	-25
$+5$	+5	+5	+5	+5	+5	+5
y	5	1	-1	-1	1	5

(4)

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8 Transformation of functions

Question 1

The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

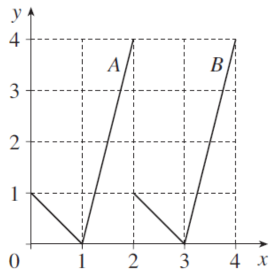
Write down an equation for the translated curve. You need not simplify your answer.

$y = (x-2)^2 - 4$

(2)

Question 2

This diagram shows graphs A and B.



(i) State the transformation which maps graph A onto graph B

A movement of 2 to the right is a translation of $\begin{pmatrix} +2 \\ 0 \end{pmatrix}$

(2)

(ii) The equation of graph A is y = f(x).
Which one of the following is the equation of graph B ?

y = f(x) + 2	y = f(x) – 2	y = f(x+2)	y = f(x-2)
y = 2f(x)	y = f(x+3)	y = f(x-3)	y = 3f(x)

$f(x-2)$

$f(x)+2$

$\uparrow$ up 2

$f(x+2) \xleftarrow{2} f(x) \xrightarrow{2} f(x-2)$

$\downarrow$ down 2

$f(x)-2$

Answer f(x-2)

(2)

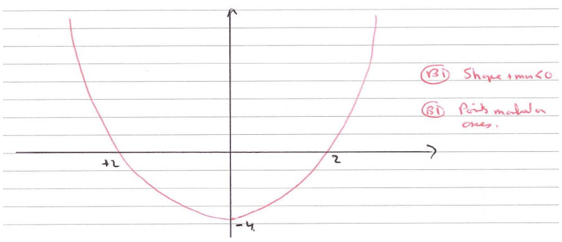
Question 3

(i) Describe the transformation which maps the curve $y = x^2$ onto the curve $y = (x+4)^2$

- Translation B1
- $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ B1 (or 4 units to the left)

(2)

(ii) Sketch the graph of $y = x^2 - 4$



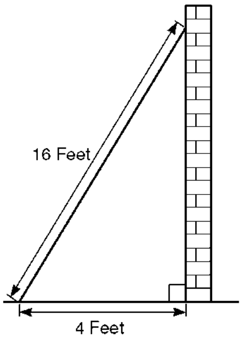
(2)

Total / 10

9 Trigonometric ratios

Question 1

Sidney places the foot of his ladder on horizontal ground and the top against a vertical wall.
The ladder is 16 feet long.



The foot of the ladder is 4 feet from the base of the wall.

(i) Work out how high up the wall the ladder reaches. Give your answer to 3 significant figures.

$\sqrt{16^2 - 4^2}$

$\sqrt{256 - 16}$ correct substitution (M1)

$\sqrt{240}$

15.49

15.5 (3sf) (A1) (2)

(ii) Work out the angle the base of the ladder makes with the ground. Give your answer to 3 sig fig

$\cos x = \frac{4}{16}$ correct ratio and substitution (M1)

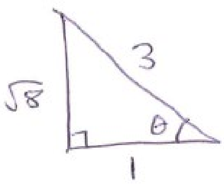
$\cos x = 0.25$

$x = 75.522$

$x = 75.5°$ (A1) (2)

Question 2

Given that $\cos \theta = \frac{1}{3}$ and θ is acute, find the exact value of $\tan \theta$

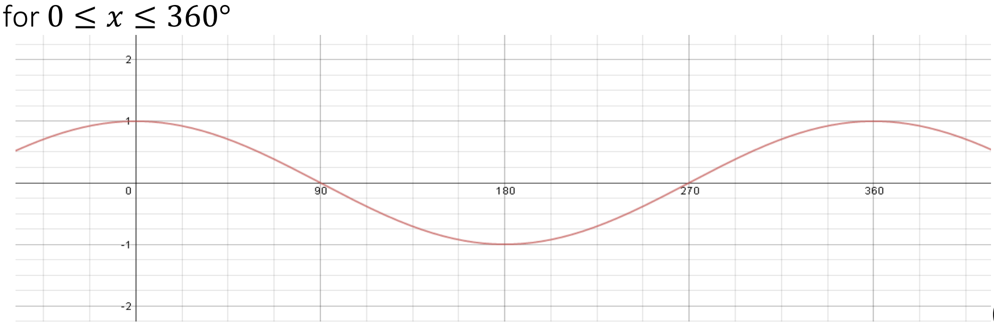


$\tan \theta = \frac{\text{opp.}}{\text{Adj}} = \frac{\sqrt{8}}{1} = \sqrt{8}.$

(3)

Question 3

Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360°$

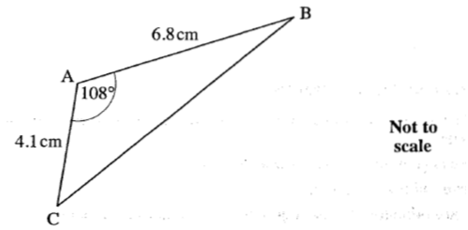


(3)

Total / 10

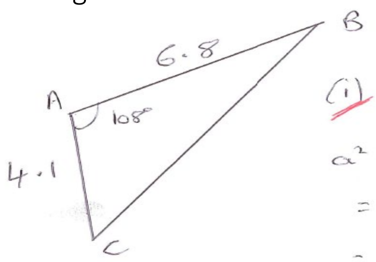
10 Sine / Cosine Rule

Question 1



For triangle ABC, calculate

(i) the length of BC



(i) By the Cosine Rule.

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 6.8^2 + 4.1^2 - 2 \times 6.8 \times 4.1 \times \cos 108$

$= 63.05 - - 17.23$

$= 80.28$

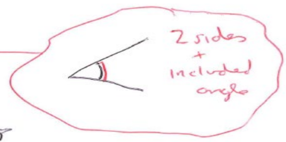
$\Rightarrow a = \sqrt{80.28} = 8.960$

(3)

(ii) the area of triangle ABC

Area of a Triangle

$= \frac{1}{2} ab \sin C$



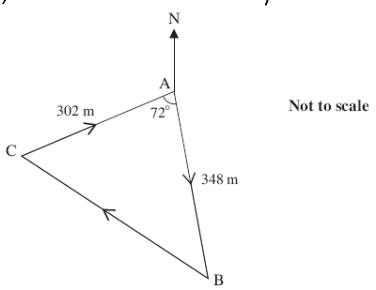
$= \frac{1}{2} \times 4.1 \times 6.8 \times \sin 108$

$= 13.26$

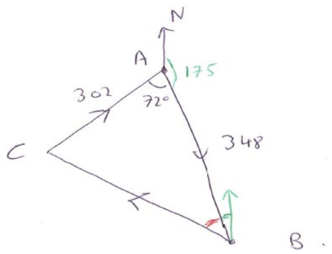
(3)

Question 2

The course for a yacht race is a triangle as shown in the diagram below. The yachts start at A, then travel to B, then to C and finally back to A.



Calculate the total length of the course for this race.



Use the Cosine Rule to find CB

$CB^2 = 302^2 + 348^2 - 2 \times 302 \times 348 \times \cos 72$

$CB = 384$

Total length = 384 + 650 = 1034m

(4)

Total / 10

11 Inequalities

Question 1

Solve

a) $x^2 - 36 \leq 0$

$$(x+6)(x-6) \leq 0$$

$-6 \leq x \leq 6$ (A1)

b) $9x^2 - 25 \geq 0$

$$(3x-5)(3x+5) \geq 0$$

$x \leq -\frac{5}{3}\ ,\ x \geq \frac{5}{3}$ (A1)

c) $3x^2 + 10x < 0$

$$x(3x+10) < 0$$

$-\frac{10}{3} < x < 0$ (A1)

(3)

Question 2

Solve $\frac{21}{x+2} - \frac{5}{x+1} < 4$

$21(x+1) - 5(x+2) < 4(x+2)(x+1)$ (M1)

$21x + 21 - 5x - 10 < 4(x^2 + 3x + 2)$

$16x + 11 < 4x^2 + 12x + 8$

$0 < 4x^2 - 4x - 3$ (M1)

$0 < (2x+1)(2x-3)$

Critical values $x = -\frac{1}{2}$ $or\ x = \frac{3}{2}$ (M1)

$x < -\frac{1}{2}$ $and\ x > \frac{3}{2}$ (A1)

....

(4)

Question 3

Solve $3x^2 - 8 > 2x$

$3x^2 - 2x - 8 > 0$

$(3x+4)(x-2) > 0$ (M1)

Critical values x = -4/3 and x=2 (M1)

$x < -\frac{4}{3}\ ,\ x > 2$ (A1)

....

(3)

Total / 10

12 **Algebraic proof**

Question 1

a) If n is a positive integer, write down expressions for the next two consecutive integers.

(n + 1) and (n + 2)   1M both correct

(1)

b) Use algebra to prove that the sum of three positive consecutive integers is always a multiple of 3.

n + n + 1 + n + 2
= 3n + 3
= 3 (n + 1)
3 is a factor so the sum is a multiple of 3

- Adding expressions and simplifying result
- Factorising
- Conclusion with reason

(3)

Question 2

Prove that the square of an odd number is also odd.

2n is an even number then 2n + 1 is an odd number

$(2n+1)^2 = 4n^2 + 4n + 1$

$4n^2 + 4n = 4(n^2 + 1)$ so this expression is a multiple of 4 hence even

so $4n^2 + 4n + 1$ is odd

- Writing algebraic expression for odd number
- Squaring expression
- Explain why result is odd

(3)

Question 3

Given that x is a positive integer, prove that $\frac{4x^3+20x}{2x^2+10}$ is always even.

$= \frac{4x(x^2+5)}{2(x^2+5)}$

$= \frac{4x}{2}$

$= 2x$ which is always even as is a multiple of 2

- Factorise
- Simplify
- Explain why result is even

(3)

Total / 10

13 Vectors

Question 1

OAP is a triangle

$\overrightarrow{OA} = 2\mathbf{f} + \mathbf{g}$ and $\overrightarrow{OB} = 3\mathbf{h}$

P is the point on AB such that AP: PB = 2:1

(a) Find the vector $\overrightarrow{BA}$ in terms of **f, g** and **h.**

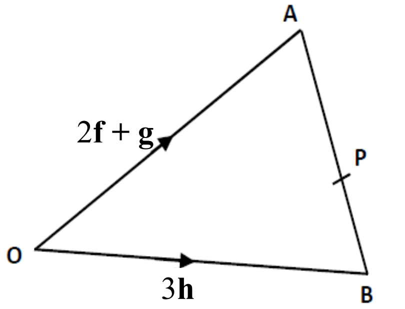


Diagram NOT drawn accurately

(1)

(b) Find the vector $\overrightarrow{PO}$ in terms of **f, g** and **h**

$\overrightarrow{PO} = \overrightarrow{PA} + \overrightarrow{AO} = \frac{2}{3}(-3\mathbf{h} + 2\mathbf{f} + \mathbf{g}) - (2\mathbf{f} + \mathbf{g})$ (M1)

$\overrightarrow{PO} = -2\mathbf{h} - \frac{2}{3}\mathbf{f} - \frac{1}{3}\mathbf{g}$ or $-\frac{1}{3}(6\mathbf{h} + 2\mathbf{f} + \mathbf{g})$ oe simplified expression (A1)

....

(2)

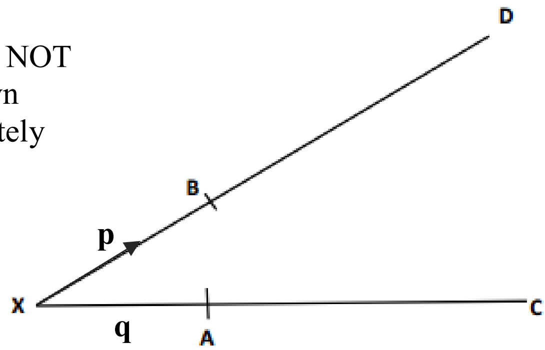
Diagram NOT drawn accurately

Question 2

B is the point on AD such that XB:BD is

A is the point on XC such that XA:XC is 1:2

$\overrightarrow{XB} = \mathbf{p}$ and $\overrightarrow{XA} = \mathbf{q}$



Use vectors to explain the geometrical relationships between the line segments BA and DC.

$\overrightarrow{BA} = -p + q$ (M1)

$\overrightarrow{DC} = -3p + 3q$ (M1)

$\overrightarrow{BA} = 3\overrightarrow{DC}$ so the lines are parallel (A1) and DC is 3 times the length of BA (A1)

(4)

Question 3

PQRS is a parallelogram.

A is the point on PR such that PA:AR is 2:1

M is the midpoint of RS.

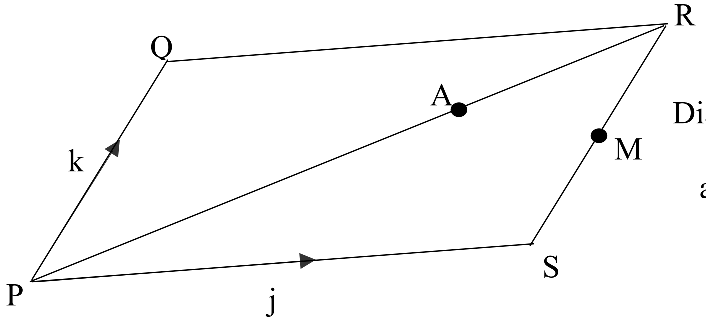


Diagram NOT drawn accurately

(b) Prove that Q, A and M are co-linear.

$\overrightarrow{QA} = -\mathbf{k} + 2/3(\mathbf{k} + \mathbf{j}) = -1/3\mathbf{k} + 2/3\mathbf{j} = 1/3\,(2\mathbf{j} - \mathbf{k})$ (M1) accept any equivalent vector

$\overrightarrow{QM} = -\mathbf{k} + \mathbf{j} + \frac{1}{2}\mathbf{k} = -1/2\,\mathbf{k} + \mathbf{j} = \frac{1}{2}(2\mathbf{j} - \mathbf{k})$ (M1) accept any equivalent vector

$\overrightarrow{QA}$ and $\overrightarrow{QM}$ are both multiples of $2\mathbf{j} - \mathbf{k}$ so are parallel and have Q as a common point so are collinear

....

(3)

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14 Probability

Question 1

A box contains 3 new batteries, 5 partly used batteries and 4 dead batteries.

Kelly takes two batteries at random.

Work out the probability that she picks two different types of batteries.

NP $\frac{3}{12} \times \frac{5}{11} = \frac{5}{44}$

ND $\frac{3}{12} \times \frac{4}{11} = \frac{1}{11}$

PN $\frac{5}{12} \times \frac{3}{11} = \frac{5}{44}$

PD $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ Multiplying each probability M1

DN $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$ Adding their probabilities M1

DP $\frac{4}{12} \times \frac{5}{11} = \frac{5}{33}$ Correct solution A1

P(two different types) = $\frac{47}{66}$

Or

NN $\frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$

PP $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ Multiplying probability of MTS by 6 M1

DD $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$ Subtracting their answer from 1 M1

Correct solution A1

P(two the same type) = $1 - \frac{19}{66} = \frac{47}{66}$

....

(3)

Question 2

Caleb either walks to school or travels by bus.

The probability that he walks to school is 0.75.

If he walks to school, the probability that he will be late is 0.3.

If he travels to school by bus, the probability that he will be late is 0.1.

Work out the probability that he will not be late.

0.75 x 0.7 = 0.525 or 0.25 x 0.9 = 0.225 M1

0.525 + 0.225 = M1

0.75 A1

....

(3)

Question 3

The two way table shows the number of deaths and serious injuries caused by road traffic accidents in Great Britain in 2013.

		Speed Limit			
		20 mph	**30 mph**	**40 mph**	**Total**
Type of Injury	**Fatal**	6	520	155	681
	Serious	420	11582	1662	13664
	Total	426	12102	1817	14345

Work out an estimate for the probability:

(a) that the accident is serious.

$\frac{13664}{14345}$ or 0.95 A1 ....

(1)

(b) that the accident is fatal given that the speed limit is 30 mph.

$\frac{520}{12102} = \frac{260}{6051}$ or 0.04

....

(1)

(c) that the accident happens at 20 mph given that the accident is serious.

$\frac{420}{13664} = \frac{15}{488}$ or 0.03 M2 (Correct working must be seen)

Allow M1 for $\frac{420}{14345} = \frac{84}{2869}$ or 0.03

....

(2)

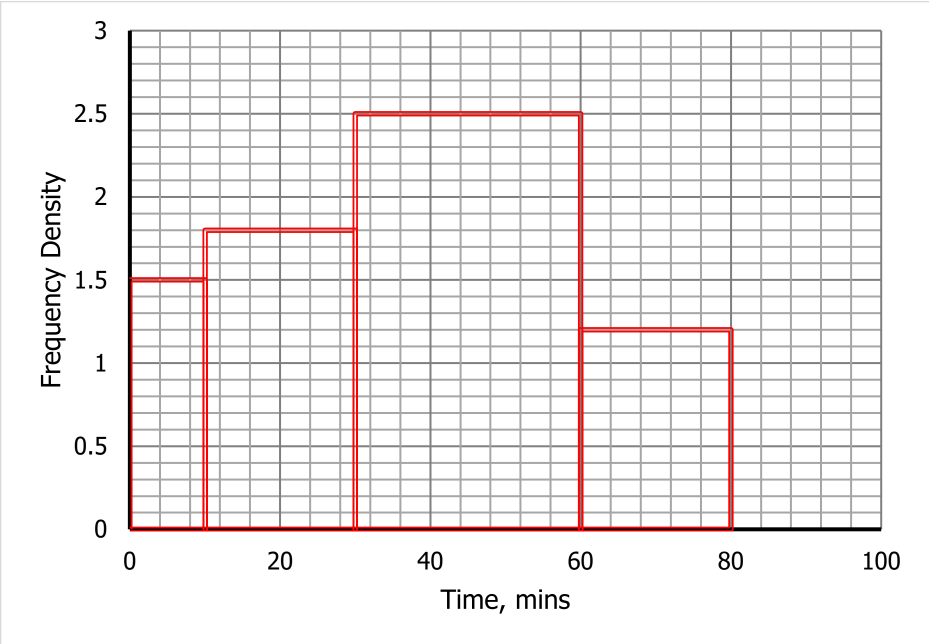
Total / 10

15 **Statistics**

Question 1

The histogram and the frequency table show some information about how much time vehicles spent in a car park.

Time, minutes			Frequency	Class Width	Freq. Density
0	$< x \leq$	10	15	10	1.5
10	$< x \leq$	30	36	20	1.8
30	$< x \leq$	60	75	30	2.5
60	$< x \leq$	80	24	20	1.2
		Total	150		



a) Use the information to complete the histogram

(2)

b) Use the histogram to find the missing frequencies in the table

1.5 x 10 = 15 B1

1.8 x 20 = 36 B1

.........15 and 36 ....

(2)

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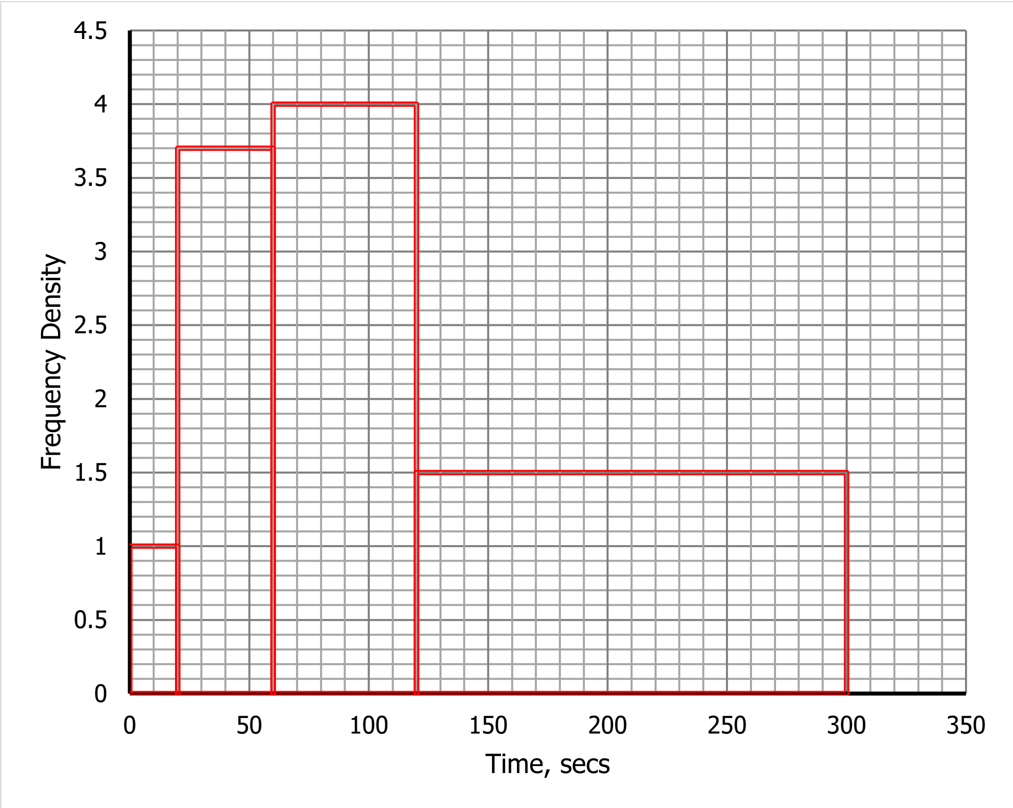
Question 2

The table shows the length of 678 phone calls made at a call centre

Time, secs	Frequency	Class Width	Freq. Density
$0 < x \leq 20$	20	20	1.0
$20 < x \leq 60$	148	40	3.7
$60 < x \leq 120$	240	60	4.0
$120 < x \leq 300$	270	180	1.5
Total	678		

a) Draw a fully labelled histogram to show the length of the phone calls.



(4)

b) Estimate the number of phone calls that lasted more than 4 minutes.

4 minutes = 4 x 60 secs = 240 secs
300 – 240 = 60 mins
60 x 1.5 = (90 calls) M1

............90 calls...............A1..............
(2)

Total / 10