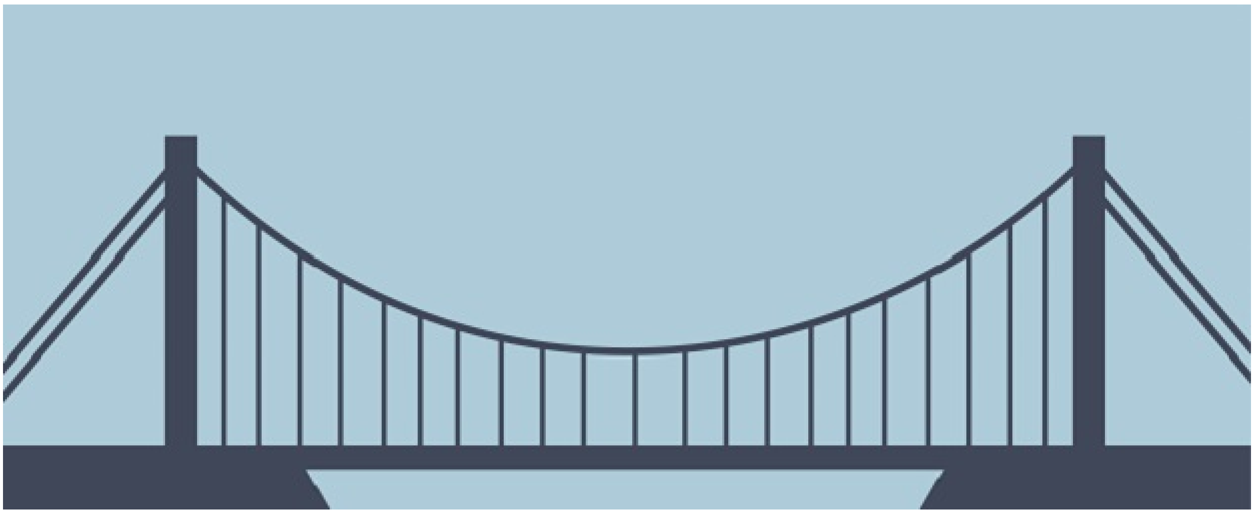
The Bridge to A level

Test Yourself Worked Solutions



1 Solving quadratic equations

Question 1

Find the real roots of the equation $x^4 - 5x^2 - 36 = 0$ by considering it as a quadratic equation in x^2

Treat as a quadratic in x^2.

Factorise $(x^2 - 9)(x^2 + 4) = 0$

→ $(x^2 - 9) = 0$ or $(x^2 + 4) = 0$

→ $x^2 = 9$ or $x^2 = -4$

→ $x = \pm 3$ or No real roots

→ $x = \pm 3$

(4)

Question 2

(i) Write $4x^2 - 24x + 27$ in the form of $a(x - b)^2 + c$

$$4x^2 - 24x + 27$$

Don't take the factor of 4 out of the constant.

$$= 4(x^2 - 6x) + 27$$

$$= 4[(x-3)^2 - 9] + 27$$

$$= 4(x-3)^2 - 36 + 27$$

$$= 4(x-3)^2 - 9$$

(4)

(ii) State the coordinates of the minimum point on the curve $y = 4x^2 - 24x + 27$.

Minimum point at (3,-9)

(2)

Total / 10

2 Changing the Subject

Question 1

Make t the subject of the formula $s = \frac{1}{2}at^2$

$$s = \frac{1}{2}at^2$$

$$2s = at^2$$

$$\frac{2s}{a} = t^2$$

$$t = \pm\sqrt{\frac{2s}{a}}$$

(3)

Question 2

Make x the subject of $3x - 5y = y - mx$

2)

$$3x - 5y = y - mx$$

$$3x + mx - 5y = y$$

$$3x + mx = y + 5y$$

$$x(3 + m) = 6y$$

$$x = \frac{6y}{(3+m)}$$

Get terms with x = terms without x.

Factorise, then divide by factor

(3)

Question 3

Make x the subject of the equation $y = \frac{x+3}{x-2}$

$$y = \frac{x+3}{x-2}$$

$$y(x-2) = x+3$$

$$xy - 2y = x + 3$$

$$xy - x = 2y + 3$$

$$x(y-1) = 2y + 3$$

$$x = \frac{2y+3}{y-1}$$

(4)

Total / 10

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3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines $x + 2y = 5$ and $y = 5x - 1$

$x + 2(5x-1) = 5$

$x + 10x - 2 = 5$

$11x = 7$

$x = \frac{7}{11}$ $y = \frac{35}{11} - 1$ $y = \frac{24}{11}$

(3)

Question 2

The lines $y = 5x - a$ and $y = 2x + 18$ meet at the point $(7,b)$. Find the values of a and b.

$5x - a = 2x + 18$

$35 - a = 14 + 18$

$a = 3$ $b = 35 - 3 = 32$

(3)

Question 3

A line and a curve has the following equations :

$3x + 2y = 7$ $y = x^2 - 2x + 3$

Find the coordinates of the points of intersection of the line and the curve by solving these simultaneous equations algebraically

Substitute y from 2nd equation into 1st.

$3x + 2(x^2 - 2x + 3) = 7$

$3x + 2x^2 - 4x + 6 = 7$

$2x^2 - x + 6 = 7$

$2x^2 - x - 1 = 0$

Factorise : $(2x+1)(x-1) = 0$

means $2x+1 = 0$ or $x - 1 = 0$

$2x = -1$ or $x = 1$

$x = -\frac{1}{2}$

when $x = -\frac{1}{2}$:

$(3 \times -\frac{1}{2}) + 2y = 7$

$-1.5 + 2y = 7$

$2y = 8.5$

$y = 4.25$

when $x = 1$

$(3 \times 1) + 2y = 7$

$3 + 2y = 7$

$2y = 4$

$y = 2$.

First point $(-\frac{1}{2}, 4.25)$

Second point (1 , 2).

(4)

Total / 10

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4 Surds

Question 1

(i) Simplify $\sqrt{24} + \sqrt{6}$

(i) $\sqrt{24} + \sqrt{6}$
$= \sqrt{4}\sqrt{6} + \sqrt{6}$
$= 2\sqrt{6} + \sqrt{6}$
$= 3\sqrt{6}$

(2)

(ii) Express $\frac{36}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers.

(ii) $\frac{36}{5-\sqrt{7}} = \frac{36}{(5-\sqrt{7})} \times \frac{(5+\sqrt{7})}{(5+\sqrt{7})}$

To rationalise the denominator of $(x - \sqrt{y})$ top + multiply by bottom $(x + \sqrt{y})$

$= \frac{36 \times (5+\sqrt{7})}{25 - 5\sqrt{7} + 5\sqrt{7} - (\sqrt{7})^2}$

$= \frac{36(5+\sqrt{7})}{18}$

$= 2(5+\sqrt{7})$

$= 10 + 2\sqrt{7}$

(3)

Question 2

(i) Simplify $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

7 i) $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$
$= 30\sqrt{6} - \sqrt{4}\sqrt{6}$
$= 28\sqrt{6}$

(2)

(ii) Express $(2 - 3\sqrt{5})^2$ in the form $a + b\sqrt{5}$, where a and b are integers.

$(2-3\sqrt{5})(2-3\sqrt{5}) = 4 - 6\sqrt{5} - 6\sqrt{5} + 9 \times 5$
$= 49 - 12\sqrt{5}$. (3)

(3)

Total / 10

5 Indices

Question 1

Find the value of the following.

(i) $\left(\frac{1}{3}\right)^{-2}$

$\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^{2}$

$= 3^2$

$= 9$

(2)

(ii) $16^{\frac{3}{4}}$

$16^{3/4} = \left(16^{\frac{1}{4}}\right)^{3}$

$= (2)^3$

$= 8$

(2)

Question 2

(i) Find a, given that $a^3 = 64x^{12}y^3$

$a^3 = 64x^{12}y^3$

$= 4^3 (x^4)^3 y^3$

$= (4x^4y)^3$

$\Rightarrow a = 4x^4y$

(2)

(ii) $\left(\frac{1}{2}\right)^{-5}$

$\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^{5}$ ← Negative index inverts the fraction

$= 2^5$

$= 32$

(2)

Question 3

Simplify $\dfrac{16^{\frac{1}{2}}}{81^{\frac{3}{4}}}$

$\dfrac{16^{\frac{1}{2}}}{81^{3/4}} = \dfrac{\sqrt{16}}{\sqrt[4]{81}^{3}} = \dfrac{4}{3^3} = \dfrac{4}{27}$

(2)

Total / 10

6 Properties of Lines

Question 1

The points A (-1,6), B (1,0) and C (13,4) are joined by straight lines. Prove that AB and BC are perpendicular.

Grad of AB = $\frac{0-6}{1--1}$ = -3

Grad of BC = $\frac{4-0}{13-1}$ = $\frac{1}{3}$

Product of gradients is = -3 x $\frac{1}{3}$ = -1. Hence AB and BC are perpendicular.

(2)

Question 2

A and B are points with coordinates (-1,4) and (7,8) respectively. Find the coordinates of the midpoint, M, of AB.

Midpoint is $(\frac{7+-1}{2}, \frac{8+4}{2})$ = (3, 6)

(1)

Question 3

A line has gradient -4 and passes through the point (2,-6). Find the coordinates of its points of intersection with the axes.

Equation of line is (y - -6) = -4(x – 2) ie y = -4x + 2

x = 0 → y = 2 Coordinates (0,14)
y = 0 → x = 0.5 Coordinates (3.5,0)

(4)

Question 4

Find the equation of the line which is parallel to y = 3x + 1 and which passes through the point with coordinates (4,5).

Gradient 3

(y – 5) = 3(x – 4)

→ y = 3x - 7

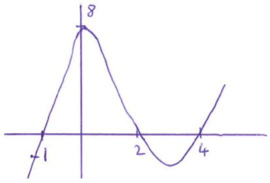
(3)

Total / 10

7 **Sketching curves**

Question 1

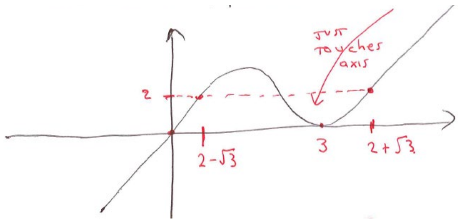
You are given that f(x) = (x + 1)(x – 2)(x – 4). Sketch the graph of y = f(x)



(3)

Question 2

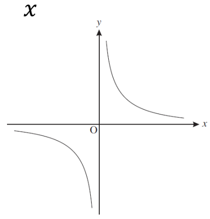
Sketch the graph of y = x(x - 3)^2



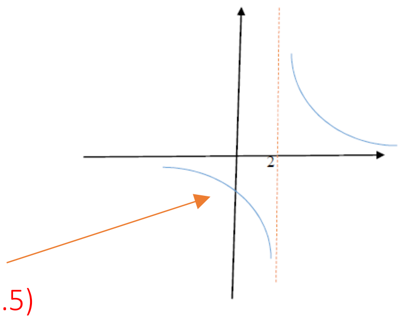
(3)

Question 3

This diagram shows a sketch of the graph of y = $\frac{1}{x}$



Sketch the graph of y = $\frac{1}{x-2}$, showing clearly any points where it crosses the axes.

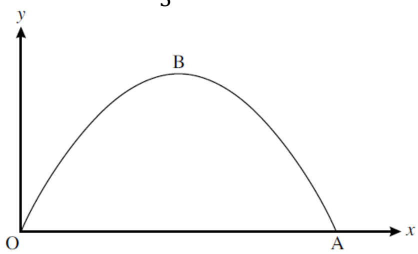


(0,-0.5)

(3)

Question 4

This curve has equation y = $\frac{1}{5}$ x (10 - x). State the value of x at the point A.



$y = \frac{x}{5}(10 - x)$

$x = 10$ B1

(1)

Total / 10

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8 Transformation of functions

Question 1

The graph of y = x2 – 8x + 25 is translated by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$. State an equation for the resultant graph.

$y = x^2 - 8x + 25 - 20$

$\Rightarrow y = x^2 - 8x + 5$

(1)

Question 2

$f(x) = x^3 - 5x + 2$

Show that $f(x - 3) = x^3 - 9x^2 + 22x - 10$

$f(x - 3) = (x - 3)^3 - 5(x - 3) + 2$

$= (x^2 - 6x + 9)(x - 3) - 5x + 15 + 2$

$= x^3 - 3x^2 - 6x^2 + 18x + 9x - 27 - 5x + 15 + 2$

$= x^3 - 9x^2 + 22x - 10$

(4)

Question 3

You are given that $f(x) = 2x^3 + 7x^2 - 7x - 12$

Show $f(x - 4) = 2x^3 - 17x^2 + 33x$

$f(x-4) = (x-4+4)(2(x-4)-3)(x-4+1)$

$= x(2x-8-3)(x-3)$

$= x(2x-11)(x-3)$

$= x(2x^2 - 11x - 6x + 33)$

$= 2x^3 - 17x^2 + 33x$

(3)

Question 4

You are given that f(x) = (x + 1)(x – 2)(x – 4). The graph of y = f(x) is translated by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

State an equation for the resulting graph. You need not simplify your answer.

(2)

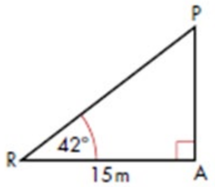
(x + 1 – 3)(x – 2 – 3)(x – 4 – 3)

ie (x -2)(x – 5)(x – 7)

Total / 10

9 Trigonometric ratios

Question 1
AP is a telephone pole. The angle of elevation of the top of the pole from the point R on the ground is 42°as seen in the diagram.



Calculate the height of the pole. Give your answer to 3 significant figures.

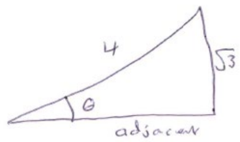
Tan 42° = $\frac{opp}{adj}$ (M1)

0.9004 = $\frac{height\ of\ pole}{15}$ (M1)

13.5(06) m = height of pole (A1)

(3)

Question 2
Given that sin Θ = $\frac{\sqrt{3}}{4}$, find in surd form the possible values of cos Θ.



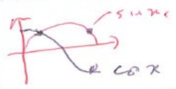
$\sin\theta = \frac{\sqrt{3}}{4}$

By Pythagoras' $4^2 = (\sqrt{3})^2 + \text{adjacent}^2$

adjacent $= \sqrt{13}$

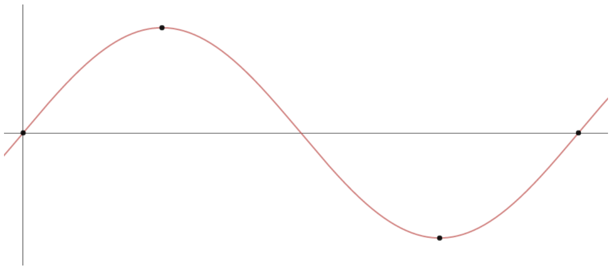
$\Rightarrow \cos\theta = \pm\frac{\sqrt{13}}{4}$

N.B. 2 values because



(3)

Question 3
The graph of *y* = sin *x* for $0 \leq x \leq 360°$ is shown below.



What are the coordinates of the 4 points labelled on the graph?

(......0..., ...0...)
(90..., ...1...)
(270..., ...-1......)
(360..., ...0...)
(4)

Total / 10

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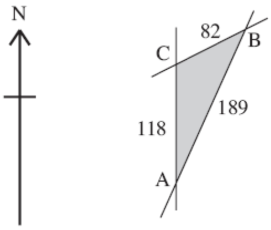
10 Sine / Cosine Rule

Question 1

This diagram shows a village green which is bordered by 3 straight roads AB, BC and CA. The road AC runs due North and the measurements are shown in metres.



Not to scale

(i) Calculate the bearing of B from C, giving your answer to the nearest 0.1°

$$\cos C = \frac{82^2 + 118^2 - 189^2}{2 \times 82 \times 118} = -0.778$$

$$C = \cos^{-1}(-0.778)$$

$$= 141.1^\circ$$

Bearing $= 180 - 141.1^\circ = 38.9^\circ$

(4)

(ii) Calculate the area of the village green.

Area $= \frac{1}{2} bc \sin C$

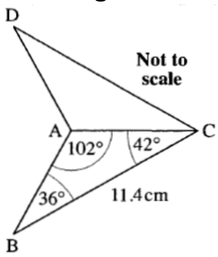
$= \frac{1}{2} \times 82 \times 118 \times \sin 141.1^\circ$

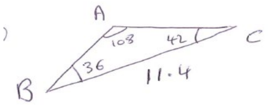
$= 3032$

(2)

Question 2

This diagram shows a logo ABCD. It is symmetrical about AC.
Find the length of AB and hence find the area of the logo





By the Sine Rule.

$$\frac{AB}{\sin 42} = \frac{11.4}{\sin 108}$$

$\Rightarrow AB = \frac{11.4 \times \sin 42}{\sin 108} = 7.798$

Area of logo $= 2 \times$ Area of Triangle

$= 2 \times \frac{1}{2} ac \sin B$

$= 11.4 \times 7.798 \times \sin 36$

$= 52.2$

(4)

Total / 10

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11 Inequalities

Question 1

Solve the inequality $x^2 < 3(x+6)$

$x^2 < 3x + 18$
$x^2 - 3x - 18 < 0$
$(x-6)(x+3) < 0$ M1
Critical values when x = 6 and x = -3 M1
$-3 < x < 6$ A1

(3)

Question 2

Solve the inequality $x^2 > 3x + 4$

$x^2 - 3x - 4 > 0$
(x-4)(x+1) >0 M1
Critical values x = 4 and x = -1 M1
x < 4 and x > 1 A1

(3)

Question 3

A rectangle has length 3x cm and width (x+2) cm. The area of the rectangle is less than 90cm. Find the range of values that x can take.

3x(x+2)<90
$3x^2 + 6x - 90 < 0$ M1
$x^2 + x - 30 < 0$
(x+6)(x-5) < 0 M1
Critical values x = -6 and x = 5
-6 < x < 5 M1
Length so x positive integer 0<x<5 A1

(4)

Total / 10

12 Algebraic proof

Question 1

a) If n is a positive integer explain why the expression 2n + 1 is always an odd number.

2n is a multiple of 2 so it must be even so 2n + 1 is the number after an even number so it must be odd.

(1)

b) Use algebra to prove that the product of two odd numbers is also odd.

(2n + 1) (2m + 1)

= 4mn + 2n + 2m + 1

= 2 (2mn + n + m) + 1

2 (2mn + n + m) must be even so

2 (2mn + n + m) + 1 must be odd because the bracket is even.

- Expand and simplify brackets
- Factorise
- Explain why factorised part is even
- State result must be odd

(4)

Question 2

a) Prove that the sum of four consecutive whole numbers is always even.

First number = n
n + (n+1) + (n+2) + (n+3) = 4n+6
=2(2n+3) which is a multiple of 2 and therefore even

(3)

b) Give an example to show that the sum of four consecutive number is not always divisible by 4.

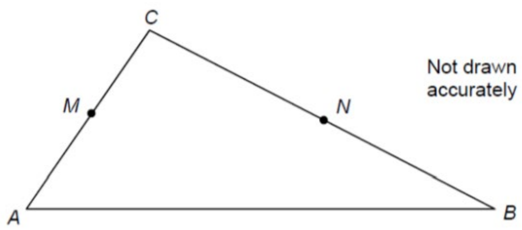
One example of many = 12+13+14+15 = 54 which is even though not divisible by 4

(2)

Total / 10

13 **Vectors**

Question 1



Triangle ABC has points M as the midpoints of AC and point N such that BN:CN = 2:3

$\overrightarrow{AM} = a$
$\overrightarrow{AB} = 2b$

a) Calculate $\overrightarrow{MN}$ giving your answer in its simplest form.

$\overrightarrow{CB} = 2b - 2a$ M1
$\overrightarrow{CN} = \frac{2}{5}\overrightarrow{CB} = \frac{2}{5}(2b - 2a)$
$= \frac{4}{5}(b - a)$ A1

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$ M1
$= a + \frac{4}{5}(b - a)$
$= \frac{4}{5}b - \frac{1}{5}a$ A1

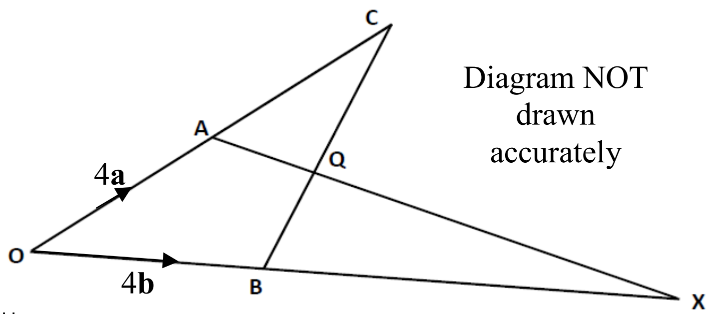
(4)

b) Are the lines MN and AB parallel? Show all of your working.

$\overrightarrow{MN} = \frac{4}{5}b - \frac{1}{5}a \neq \lambda b$ *for any value of* λ *and so the lines are not parallel*

(1)

Question 2



In the diagram

$\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 4\mathbf{b}$
A is the midpoint of OC
BQ:QC = 1:2
Find, in terms of **a** and **b**, the vector that represents

(a) $\overrightarrow{BC}$

$\overrightarrow{BC} = -4\mathbf{b} + 8\mathbf{a}$ or $4(2\mathbf{a} - \mathbf{b})$ oe (B1)

....

(1)

(b) $\overrightarrow{AQ}$

$\overrightarrow{AQ} = -4\mathbf{a} + 4\mathbf{b} + \frac{1}{3}(-4\mathbf{b} + 8\mathbf{a})$ (M1)

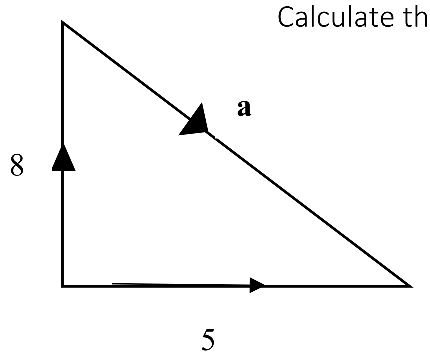
$\overrightarrow{AQ} = -\frac{4}{3}\mathbf{a} + \frac{8}{3}\mathbf{b}$ or $\frac{2}{3}(4\mathbf{b} - 2\mathbf{a})$ oe (A1)

....

(2)

Question 3

Calculate the magnitude of vector **a**.



$Vector\ \boldsymbol{a} = \binom{5}{8}$

$\boldsymbol{a}^2 = 5^2 + 8^2 = 25 + 64$ M1

$|\boldsymbol{a}| = \sqrt{89} = 9.43$ A1

(2)

Total / 10

14 Probability

Question 1

Laura has 9 tins of soup in her cupboard, but all the labels are missing.

She knows that there are 5 tins of tomato soup and 4 tins of vegetable soup.

She opens three tins at random.

Work out the probability that she opens more tins of vegetable soup than tomato soup.

TVV $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$	Correct outcomes chosen	M1
VTV $\frac{4}{9} \times \frac{5}{8} \times \frac{3}{7} = \frac{5}{42}$	Multiplying each probability	M1
VVT $\frac{4}{9} \times \frac{3}{8} \times \frac{5}{7} = \frac{5}{42}$	Adding their probabilities	M1
VVV $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21}$	Correct solution	A1

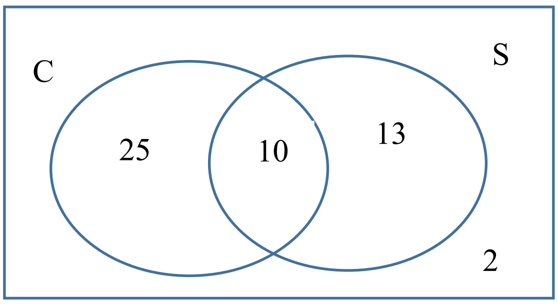
P(more vegetable) = $\frac{17}{42}$

....
(4)

Question 2

A summer camp runs coasteering and surfing classes.
50 children attend the camp
35 children do coasteering
10 children do both classes
2 children do neither class

a) Draw a Venn diagram to represent this information



(2)

A child attending the summer camp is selected at random.

b) Find the probability that the child

i) did exactly one class

25+13 = 38

38/50 = 19/25

……………………………………… (2)

ii) did surfing, given that they did not do coasteering

13/15

……………………………………… (2)

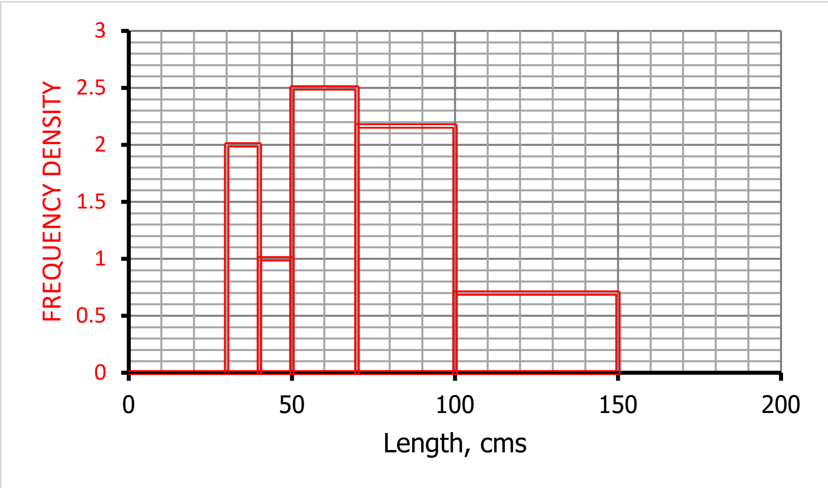
Total / 10

15 Statistics

Question 1

The table and histogram show the lengths of some pythons.

Length, cms	Frequency	Class Width	Freq. Density
30 < x ≤ 40	20	10	2.0
40 < x ≤ 50	10	10	1.0
50 < x ≤ 70	50	20	2.5
70 < x ≤ 100	65	30	2.167
100 < x ≤ 150	35	50	0.7
Total			



(a) Use the histogram to find the missing frequencies in the table

50 x 0.7 = (35) M1 for at least one calculation

30 x 2.167 = (65) A1 for both correct

....

(2)

(b) Estimate the median python length.

(181 + 1)/2 = 91

First three bars = 80, need 11 more for median. M1

11 / 2.2 = 5 M1

70 + 5 = 75 cms

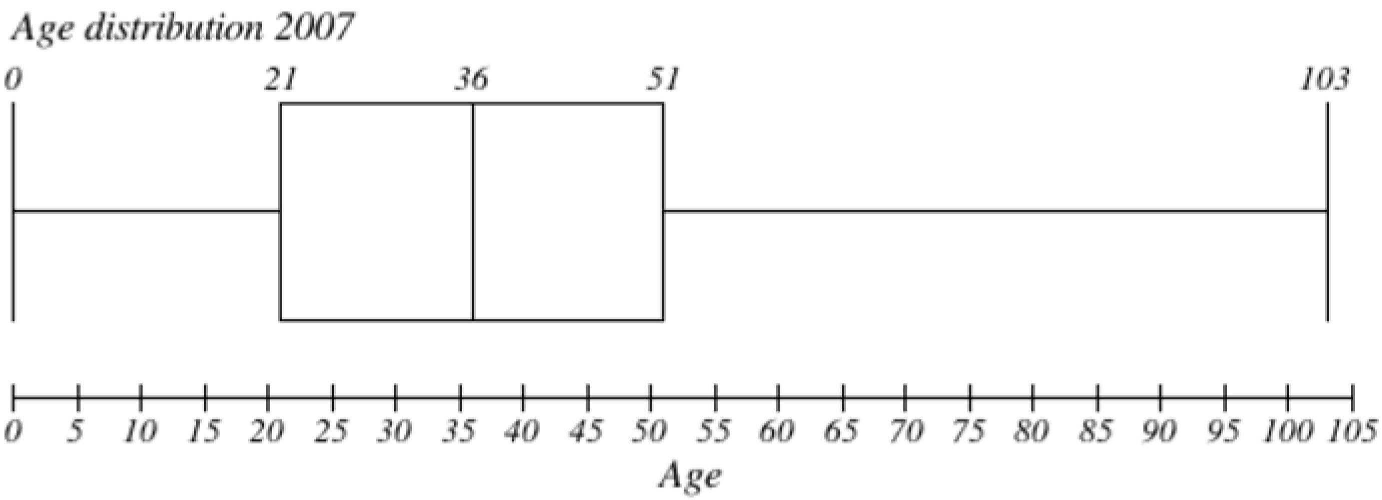
......75cms...............A1.................

(3)

Question 2

a) In France in 2007 25% of the population were under 21years old. 50% were under 36. The interquartile range of the ages was 30 years. The oldest person was 103 years old.

Show this information on a boxplot



(3)

b) It is predicted that by 2040 the age distribution in France will have a lower quartile of 26 years, a median of 44 years and an upper quartile of 66 years.

Make two comments about the predicted change in the age distribution from 2007 to 2040.

In 2007, corresponding values are 21, 36, 51

Comment 1

Refer to increased median and include meaning e.g. people on average will be older in 2040

Comment 2

Refer to IQR – it is lower in 2007 showing more consistency in ages

(2)

Total / 10